Channel Estimation Techniques for Diffusion-Based Molecular Communications

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***Abstract*—In molecular communication (MC) systems, the *expected* number of molecules observed at the receiver over time after the instantaneous release of molecules by the transmitter is referred to as the channel impulse response (CIR). Knowledge of the CIR is needed for the design of detection and equalization schemes. In this work, we present a training-based CIR estimation framework for MC systems which aims at estimating the CIR based on the *observed* number of molecules at the receiver due to emission of a *sequence* of known numbers of molecules by the transmitter. In particular, we derive maximum likelihood (ML) and least sum of square errors (LSSE) estimators. We also study the Cramer Rao (CR) lower bound and training sequence design for the considered system. Simulation results confirm the analysis and compare the performance of the proposed estimation techniques with the CR lower bound.**

I. State of Art

The design of any communication system crucially depends on the characteristics of the channel under consideration. In MC systems, the impact of the channel on the number of observed molecules can be captured by the channel impulse response (CIR) which is defined as the *expected* number of molecules counted at the receiver at time t after the instantaneous release of a known number of molecules by the transmitter at time $t = 0$. The CIR, denoted by $\bar{c}(t)$, can be used as the basis for the design of equalization and detection schemes for MC systems. In most existing works on MC, the CIR is assumed to be perfectly known for receiver design [1], [2]. In practice, the CIR has to be estimated. One widely employed approach in the literature for determining the CIR is as follows [3]

$$\bar{c}(t) = \iiint_{\mathbf{a}\in V^{\text{rec}}} \bar{\mathcal{C}}(\mathbf{a}, t)\mathrm{d}a_x\mathrm{d}a_y\mathrm{d}a_z, \tag{1}$$

where V^{rec} is the receiver volume and $\bar{\mathcal{C}}(\mathbf{a}, t)$ is the average concentration of the molecules at a given coordinate $\mathbf{a} = [a_x, a_y, a_z]$ and at time t after release by the transmitter. However, this approach may not be applicable in many practical scenarios. First, the CIR can be obtained based on (1) only for the special case of a fully *transparent* receiver where it is assumed that the molecules move through the receiver as if it was not present in the environment. However, for general receivers, the relationship between the concentration $\bar{\mathcal{C}}(\mathbf{a}, t)$ and the number of observed molecules $\bar{c}(t)$ may not be as straightforward. Second, solving the differential equation associated with Fick's second law to find $\bar{\mathcal{C}}(\mathbf{a}, t)$ is possible only for simple and idealistic environments. Finally, even if an expression for $\bar{\mathcal{C}}(\mathbf{a}, t)$ can be obtained for a particular MC system, it will be a function of several channel parameters such as the distance between the transmitter and the receiver and the diffusion coefficient. However, in practice, these parameters may not be known a priori and also have to be estimated [4]. This complicates finding the CIR based on $\bar{\mathcal{C}}(\mathbf{a}, t)$.

II. Main Idea of Proposed CIR Acquisition

Fortunately, for most communication problems, including equalization and detection, only the *expected* number of molecules that the receiver observes at the sampling times is needed [1], [2]. Therefore, knowledge of how the average concentration is related to the channel parameters is not required, and hence, the difficulties associated with deriving $\bar{\mathcal{C}}(\mathbf{a}, t)$ can be avoided by directly estimating the CIR. In this work, we develop a training-based CIR estimation framework which enables the acquisition of the CIR based on the observed number of molecules at the receiver due to emission of a sequence of known numbers of molecules by the transmitter. To the best of the authors' knowledge, this problem has not been studied in the MC literature, yet.

In contrast to MC, for conventional wireless communication, there is a rich literature on channel estimation, mainly for linear channel models and impairment by additive white Gaussian noise (AWGN), see [5], and the references therein. Channel estimation was also studied for non-linear and/or non-AWGN channels especially in optical communication. For example, for the photon-counting receiver, a linear time-invariant channel model with Poisson noise was considered in [6] and a non-linear channel model with Poisson noise was investigated in [7]. However, the MC channel model considered in this poster is neither linear nor impaired by AWGN and is also different from that in [7]. Therefore, the results known from conventional wireless communication are not directly applicable to MC.

III. Key Results

For the statement of the results, we use the following definitions and notations: Due to the memory of the MC channel, inter-symbol interference (ISI) occurs [1]. Here, we assume a MC channel with L memory taps where $\bar{\mathbf{c}}$ and $\hat{\bar{\mathbf{c}}}$ denote the actual and estimated CIR vectors, respectively. Moreover, let K denote the adopted training sequence length. In order to compare the performances of the considered estimators quantitatively, we define the normalized mean and variance of the estimation error $\mathbf{e} = \hat{\bar{\mathbf{c}}} - \bar{\mathbf{c}}$ as

$$\overline{\text{Mean}}_{\mathbf{e}} = \frac{\|\mathbb{E}\left\{\mathbf{e}\right\}\|^2}{\|\mathbb{E}\left\{\bar{\mathbf{c}}\right\}\|^2} \text{ and } \overline{\text{Var}}_{\mathbf{e}} = \frac{\mathbb{E}\left\{\|\mathbf{e}\|^2\right\} - \|\mathbb{E}\left\{\mathbf{e}\right\}\|^2}{\|\mathbb{E}\left\{\bar{\mathbf{c}}\right\}\|^2}, \tag{2}$$

respectively, where $\mathbb{E}\{\cdot\}$ denotes expectation and $\|\cdot\|$ denotes the norm of a vector.

Result 1: The maximum likelihood (ML) and least sum of square errors (LSSE) CIR estimators for the considered MC system are biased in general. However, it can be shown that both the ML and LSSE estimators are asymptotically unbiased. The asymptotic unbiasedness of the proposed estimators is numerically verified in Fig. 1[1] where the normalized mean of

[1]The detailed description of the parameters used for the simulation results is not provided here due to space constraints.

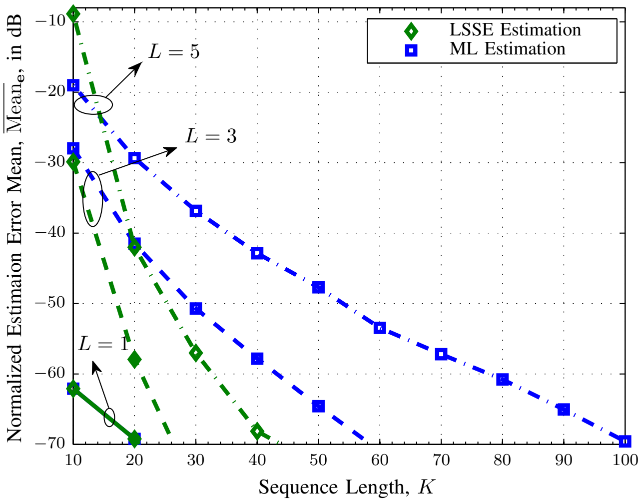

Fig. 1. Normalized estimation error mean, $\overline{\text{Mean}}_{\mathbf{e}}$, in dB vs. the training sequence length, K, for $L \in \{1, 3, 5\}$.

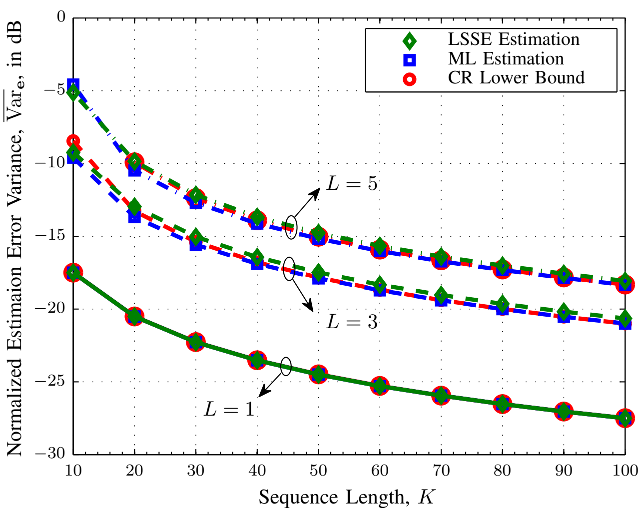

Fig. 2. Normalized estimation error, $\overline{\text{Var}}_{\mathbf{e}}$, in dB vs. the training sequence length, K, for $L \in \{1, 3, 5\}$.

the estimation error, $\overline{\text{Mean}}_{\mathbf{e}}$, in dB is plotted vs. the training sequence length, K, for $L \in \{1, 3, 5\}$. Additionally, from Fig. 1, we observe that the error mean increases as the number of channel taps increases.

Result 2: The CR bound is a lower bound on the variance of any unbiased estimator of a deterministic parameter. However, the ML and LSSE estimators are biased in general. Hence, the error variances of the ML and LSSE estimates may fall below the CR bound. However, as $K \to \infty$, the ML and LSSE estimators become asymptotically unbiased, cf. Result 1, and the CR bound becomes a valid lower bound. To show this, in Fig. 2, we plot the normalized estimation error variance, $\overline{\text{Var}}_{\mathbf{e}}$, in dB vs. the training sequence length, K, for $L \in \{1, 3, 5\}$. For large K when the CR bound is valid, the error variance of the ML estimator coincides with the CR bound and the error variance of the LSSE estimator is very close to the CR bound. This reveals the effectiveness of the proposed estimators.

Result 3: The LSSE estimator employs a linear filter to compute $\bar{\mathbf{c}}$ whereas for the ML estimator, solving a system of nonlinear equations is required. We note that since the training sequence is fixed, the linear LSSE filter can be calculated offline and then be used for online CIR estimation. Therefore, the

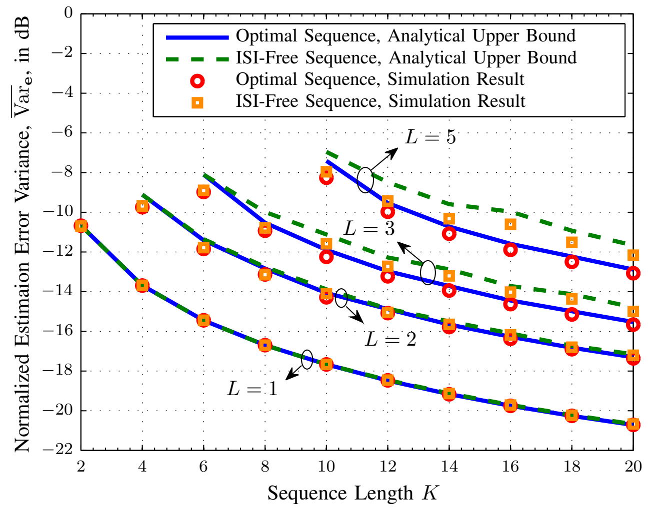

Fig. 3. Normalized LSSE estimation error, $\overline{\text{Var}}_{\mathbf{e}}$, in dB vs. the training sequence length, K, for $L \in \{1, 2, 3, 5\}$.

calculation of $\hat{\hat{\mathbf{c}}}$ for the LSSE estimator is considerably less computationally complex than the computation of $\hat{\hat{\mathbf{c}}}$ for the ML estimator. Moreover, the results in Fig. 2 suggest that the simple LSSE estimator provides a favorable complexity-performance tradeoff for CIR estimation in the MC system.

Result 4: We present two different training sequence designs for CIR estimation in MC systems: *i)* An optimal training sequence design which minimizes an *upper bound* on the *average* estimation error for the LSSE estimator, and *ii)* a suboptimal ISI-free training sequence where the transmitter emits molecules only once every L symbol intervals in order to avoid ISI during estimation. The results are given in Fig. 3 where we show the normalized LSSE estimation error, $\overline{\text{Var}}_{\mathbf{e}}$, in dB vs. the training sequence length, K, for $L \in \{1, 2, 3, 5\}$. We observe from Fig. 3 that the performance of the ISI-free sequence coincides with that of the optimal sequence for all sequence lengths when $L = 1$, and for $L > 1$, the difference between the error variances of the ISI-free sequence and the optimal sequence increases as L increases. This result suggests that for MC channels with small numbers of taps, a simple ISI-free training sequence is a suitable option.

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